AVOIDING SYMBOLOGY

PENN SUMMER PREP PROGRAM
EXPLORATIONS IN MATHEMATICAL INQUIRY

MATT DECROSS

Both veteran and novice mathematicians often make the mistake of overusing symbols when a simple English explanation would be more enlightening. Reasons for doing this include trying to make your work sound more technically impressive or masking the fact that you don't entirely understand your own work. In the long run, both the author and the reader will get more out of a paper where the ideas are presented as clearly as possible by only using symbols where they express an idea more clearly than otherwise.

Keeping the above in mind, translate the following statements into plain English:

(1)

$$\forall\, r \in \mathbb{R}, \forall\, \epsilon > 0,\ \exists\, (a,b) \in \mathbb{Z}^2 : |r - \frac{a}{b}| < \epsilon$$

(2)

$$\forall\, S \subseteq \mathbb{Z}, (\exists\, (a,b) \in \mathbb{Z}^2 : a > x\ \forall\, x \in S, b < y\ \forall\, y \in S) \implies (\exists\, C \in \mathbb{Z} : |S| = C)$$

(3)

$$\exists\, S \subseteq \mathbb{R} : (\nexists\, C \in \mathbb{R} : |S| < C) \text{ and } \nexists\, ((a,b) \subset \mathbb{R} : (a,b) \subseteq S)$$

For reference, a list of most of the symbols used above and their translations is provided below. See also `https://en.wikipedia.org/wiki/List_of_mathematical_symbols` for more.

$\forall$	for all/every/each
$\subseteq$	subset of, contained in
$\subset$	(proper) subset of
$\mathbb{R}$	the set of all real numbers
$\mathbb{Z}$	the set of all integers
$\in$	element of
$\exists$	there exists
: or $\|$	such that
$\implies$	implies
$\|\ \|$	absolute value, or the number of elements in a set

Putting a slash through any of the above symbols usually adds the words "does not" in front of the definition.